NEW THEORETICAL METHOD
FOR ESTABLISHING INDENTATION ROLLING RESISTANCE

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Abstract: Theoretical analysis indicates an underestimation of the calculated values of resistance of conveyor belts in relation to the measurement results. Disproportions are mainly observed for large mass capacities of conveyors. In this paper previous method for calculations of indentation rolling resistance was improved. Indentation rolling resistance is the main component of motion resistance for conveyors longer than 80 m. The most important impact on rolling resistance have damping factor of belt. This parameter was included in calculations in the new approach, moreover, a defined way of its determination in laboratory conditions was indicated. Another important parameter is the modulus of elasticity affecting the transverse rigidity of the belt. By analyzing elastic energy in bottom cover and cord of the belt new equations (which included construction and belt type) were established. In addition, the impact of the distribution of the loads along the idler on the rolling resistance value was analyzed. On this basis different equations for calculations of center and side idler was proposed. New theoretical model was verified in both laboratory test and measurements conducted in mine conditions.

A series of tests of rolling resistance and damping properties were carried out for belts of various core designs and various rubber of bottom covers. Damping factors and modules of elasticity established in laboratory conditions were applied to new calculations algorithms. Obtained results were compared with measured values. Final verification of the new theoretical model was a comparison of calculated resistance with measured values for conveyor in mine. The obtained results have much higher convergence than the previous calculations. The new calculations allows for more accurate dimensioning of the drive and other elements of the belt conveyor.

Keywords: *belt conveyor, conveyors belt on idler indentation rolling resistance, motion resistance*

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1. INTRODUCTION

Belt conveyors are the main means of transporting the bulk materials in mines and in many other branches of industries (Konieczna-Fuławka 2019). They are also used in unconventional ways, e.g., to transport people (Uth et al. 2019). Despite they are commonly used, they still generate high energy consumption due to motion resistance.

Belt indentation rolling resistances may account for up to 60% of main resistances, which are the dominant component of motion resistances in conveyors longer than 80 m. Indentation rolling resistances may be influenced not only by belt design parameters, but also by belt properties. Existing methods differ in how they allow for belt damping parameters (Jonkers 1980; Spaans 1991; Lodewijks 1995). In addition, no uniform and clearly defined method exists for determining belt parameters which influence the scale of belt indentation rolling resistance. Interaction between the belt and the idler effects compressive stresses in the bottom cover and only partly in the belt core (Qiu 2006; Wheeler and Munzenberger 2009). Therefore, current measurement methods are ineffective in accurately reflecting the actual conditions, as indentation occurs not only in the bottom cover and in the core, but also in the top cover (Wheeler and Munzenberger 2011). Measurements performed on a belt conveyor in actual operating conditions (Fig. 1) indicate that the analytical method for calculating resistances to motion
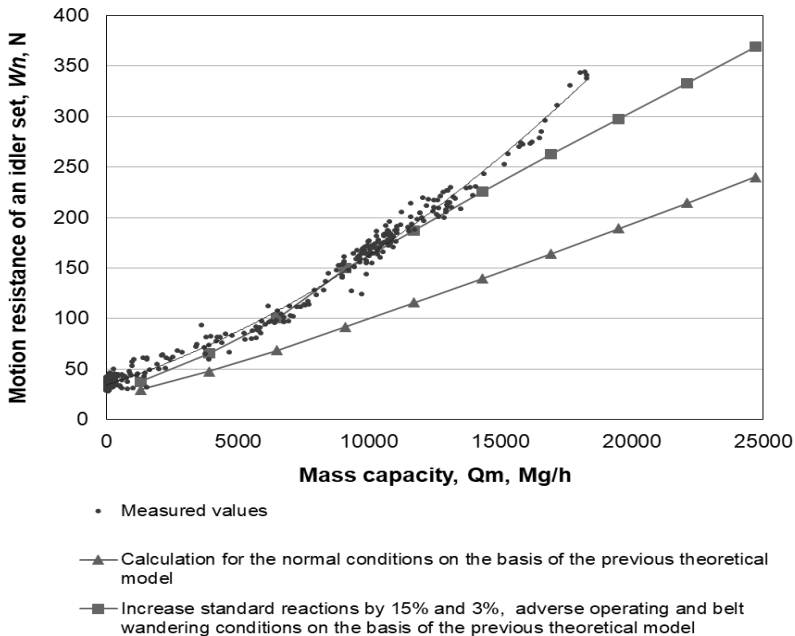

Fig. 1. Relationship between the resistance to motion of an idler set and mass capacity
(Kisielewski 2016; Konieczna-Fuławka 2019]

(including belt indentation rolling resistances), as developed at Wroclaw University of Science and Technology, offers underestimated results (Hardygóra et al. 2009; Gładysiewicz 2003).

The graph in Fig. 1 shows the relationship between mass capacity and the motion resistances of a complete idler set calculated from the measurements. The thin line indicates the function which approximates the measurement results (dots). In the case of limited mass capacities, the calculation results overlap with the measurement results. With mass capacity increasing to above 10 000 Mg/h, the disproportion begins to rise, and the results indicate that the calculations performed for standard reactions and for actual idler characteristics (triangles) are underestimated with respect to the measured values of motion resistances in an idler set. By increasing standard reactions by 15% and 3%, as well as by allowing for default idler characteristics and adverse operating conditions, i.e. for substantial belt wandering (squares) in the calculation model, it is possible to bring the calculated values close to the measurement results. The underestimation is however unfavourable for almost half of the measured values, and most importantly for the values observed at high mass capacities (above 11 000 Mg/h) Importantly, a belt conveyor showing substantial belt wandering cannot operate at its maximum capacity, and therefore the calculations for these variants are only hypothetical.

2. NEW CALCULATION MODEL

2.1. DAMPING FACTOR

The key issue when determining belt indentation rolling resistance is to find damping function $F(\delta)$. Indentation rolling resistance calculation methods in accordance with models (Jonkers 1980; Spaans 1991) introduce phase lag angle δ:

$$w_e = F(\delta) \cdot \sqrt[3]{\frac{q_T^4}{D_k^2 \cdot \lambda \cdot c_e}} \,, \tag{1}$$

where:

D_k – idler diameter, m,
q_T – unit load, N/m,
λ – bending coefficient of the belt, –,
c_e – artificial belt transverse stiffness, N/m³.

In contrast, the method developed at Wroclaw University of Science and Technology (Gładysiewicz and Konieczna 2018) employs the damping factor based on the hysteresis loop. The analysis of the theoretical model allows a relationship to be found between damping factor ψ_e and phase lag angle δ. Damping function $F(\delta)$ may be also

expressed as the product of damping factor ψ_e and constant C, which can be determined from theoretical analyses; in such case unit indentation rolling resistance is:

$$w_e = C \cdot \psi_e \sqrt[3]{\frac{q_T^4}{D_k^2 \cdot \lambda \cdot c_e}}. \tag{2}$$

Both the damping factor and the modulus of elasticity at compression may be determined on the basis of the hysteresis loop (Fig. 2).
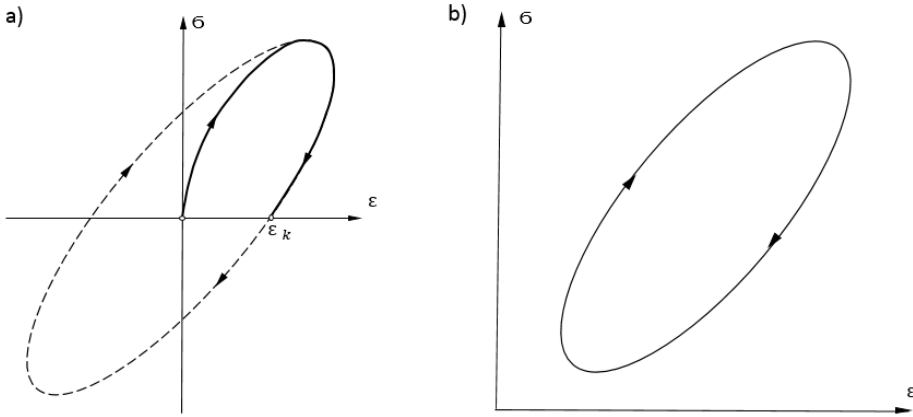

Fig. 2. Hysteresis loop:
a) for the theoretical model, b) established in laboratory tests

The hysteresis loop designated with the continuous line in Fig. 2b adequately describes the cyclical process of belt compression on idlers. Most authors investigating belt indentation rolling resistances (Jonkers 1980; Spaans 1991) use a loop typical of continuous, cyclical harmonic loads. Such a loop is designated with the dashed line in Fig. 2a. However, measurement problems occur in laboratory tests, in which attempts are made at simulating discontinuous loads in order to achieve a hysteresis loop adequate to the process of cyclical compression.

As it is impossible to investigate a hysteresis loop adequate to actual conditions (Fig. 2a), i.e., a loop which starts from zero load and zero belt strain, calculations should be based on a loop with non-zero initial load (Fig. 2b). Theoretical relationships must be therefore identified between the damping factors for both loop variants shown in Fig. 2. The relationship between the laboratory-measured damping factors ψ_{lab} and the damping factor adequate to actual conditions ψ_{th} (Konieczna-Fuławka 2019) is described in the following equation:

$$\psi_{th} = -1.0385\psi_{lab}^2 + 2.0334\psi_{lab} - 0.0021. \tag{3}$$

Equation (3) may be used to find an analytical relationship between the belt indentation rolling resistance based on the laboratory-determined damping factor (Fig. 3) (Gładysiewicz and Konieczna 2018).
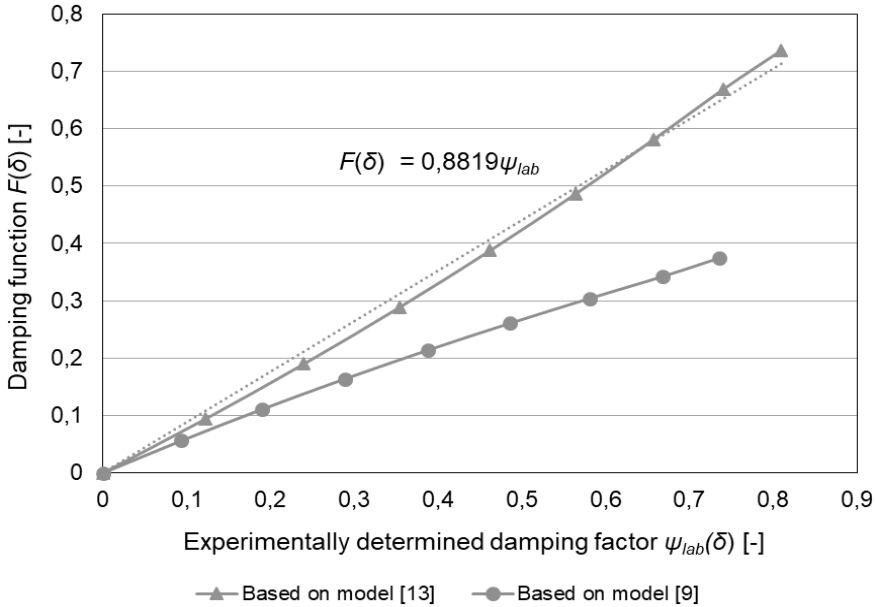

Fig. 3. Relationship between the damping function and the experimentally determined damping factor

After Eq. (1) is completed with the relationship of Fig. 3, a corrected equation for unit indentation rolling resistance is obtained:

$$w_e = 0.882 \cdot \psi_{lab} \cdot \sqrt[3]{\frac{q_T^4}{D_K^2 \cdot \lambda \cdot c_e}}. \tag{4}$$

2.1. DAMPING FACTOR

Calculating artificial unit belt transverse stiffness requires analysing the distribution of compressive loads in both the pulley cover and the belt core. In actual conditions, compressive loads cease and the top cover is not involved in the cyclic compression process. For the purpose of calculating rolling resistances, compression process in both the bottom cover and the core must be analysed separately for multiply textile belts and for steel-cord belts. Depending on belt design, the distribution of compressive loads also changes in the core rubber. In this case, core type is the decisive factor. In the case of a textile belt, the distribution of compressive loads determined in labora-

tory tests remains uniform across the thickness of the sample, while in actual conditions the changes of stress in the belt core are stepwise (Fig. 4).
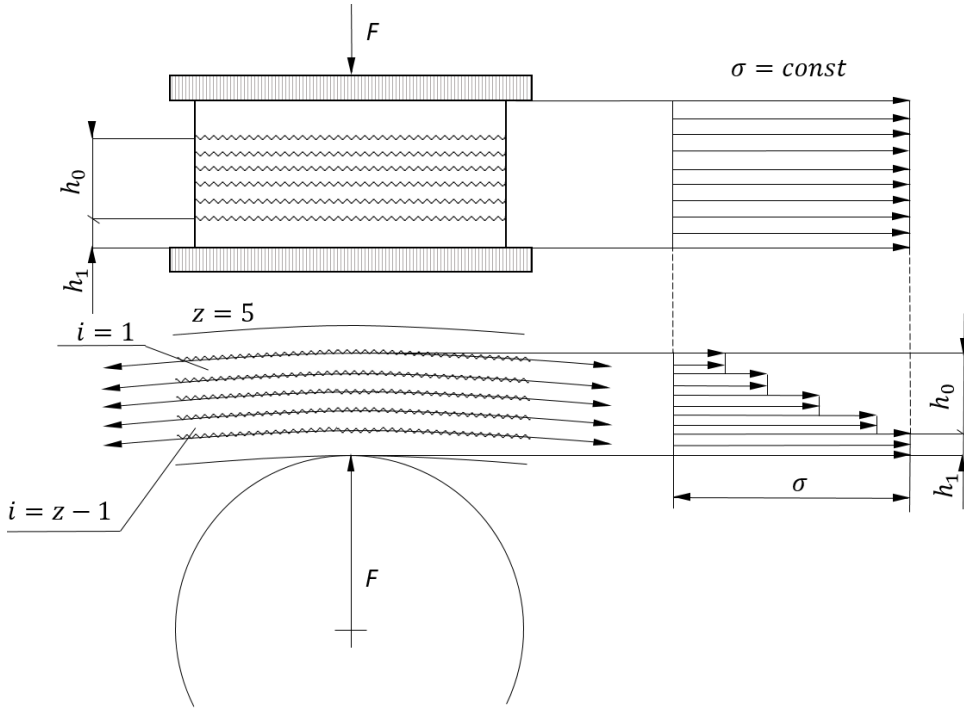

Fig. 4. Hypothetical distribution of compressive loads in a textile belt (Gładysiewicz 1990)

Artificial textile belt transverse stiffness is (Gładysiewicz 1990):

$$c_e = \frac{E_1 \cdot E_0}{h_1 \cdot E_0 + \lambda_z \cdot h_0 \cdot E_0},$$ (5)

where:

E_1 – modulus of elasticity of cover rubber, Pa,
E_0 – modulus of elasticity of core rubber, Pa,
h_1 – bottom cover thickness, m,
h_0 – core thickness, m,
λ_z – factor depending on the number of plies in the belt, –.

Artificial belt transverse stiffness is determined in laboratory conditions by assuming uniform stress distribution across the whole belt sample (Fig. 4). In order to reflect the processes which occur in actual operating conditions, a sample cut from the belt should be prepared by removing the top cover, as it is not involved in the compression process during belt operation.

In the case of a steel cord belt, a compressive load distribution of Fig. 5 may be assumed.
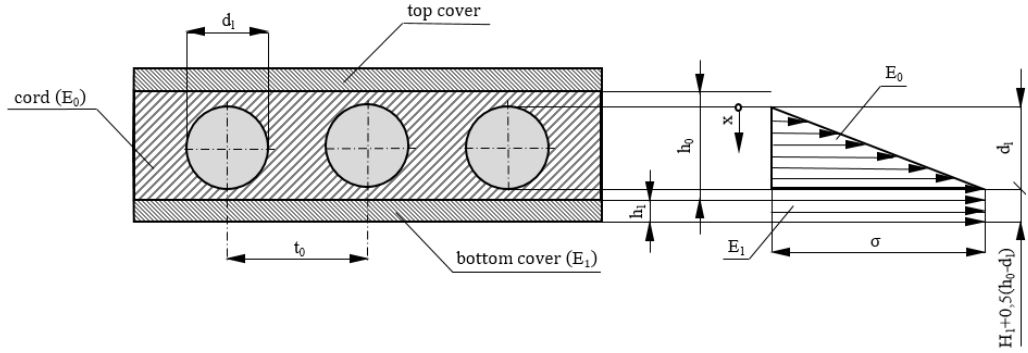

Fig. 5. Hypothetical distribution of compressive loads in a steel-cord belt

$$c_e = \frac{E_1 \cdot E_0}{E_0 \cdot h_1 + E_1 \cdot \left(\frac{1}{2} h_0 - 0.284 \cdot d_l \right)}. \tag{6}$$

As in the case of textile belts, artificial transverse stiffness in steel-cord belts depends on the modulus of elasticity for the cover rubber E_1 and for the core rubber E_0, as well as on bottom cover thickness h_1, core thickness h_0 and cord diameter d_l.

2.3. INFLUENCE OF LOAD DISTRIBUTION

Current tests (Król and Kisielewski 2014; Liu et al. 2019) and theoretical analyses (Kessler 1986) indicate that loads have a significant influence on belt indentation rolling resistance. Interestingly, however, this influence may be due to not only the value of the force, but also its distribution along the generator of the idler.

The indentation rolling resistance of the belt on an idler set is the sum of the resistances on each of the idler rollers. Finding the rolling resistance for an individual idler roll requires integrating relationship (4) over the length of the linear contact zone between the belt and the idler roll x, as measured along the generator of the idler roll. Unit idler load q_T is the parameter which changes its value along the contact zone, and therefore finding the rolling resistance for an individual idler roll requires calculating the following integral:

$$W_e = \int_0^x w_e \cdot dx = 0.882 \cdot \frac{\psi_{lab}}{\sqrt[3]{D_K^2 \cdot \lambda \cdot c_e}} \int_0^x (q_T)^{\frac{4}{3}} \cdot dx. \tag{7}$$
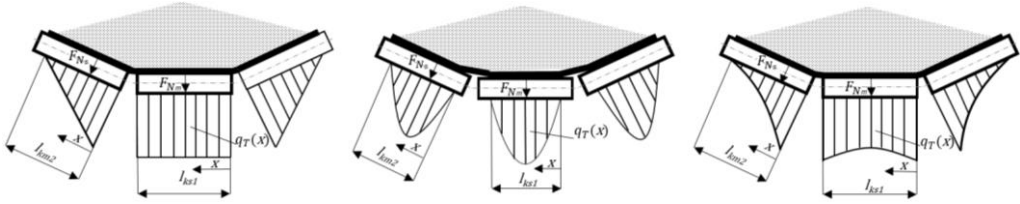
Fig. 6. Load distribution along the contact zone of idlers with the belt:
a) according to models with maximum values observed on the margin of the contact zone (Jennings 2014),
b) distribution corrected by laboratory measurements (Gładysiewicz 2003),
c) according to the theoretical model for bulk solids (Krause and Hettler 1974)

The analysis of relationship (7) indicates that key issues include finding the length of contact zones between the belt and both the side idlers and the middle idler, as well as the load distributions along those zones. In the case of a three-idler upper set, the length of the contact zone as measured along the idler generator may be calculated from empirical equations derived from measurements (Gładysiewicz 2003). The length of the contact zone between the belt and the idler has itself no influence on the value of the calculated rolling resistance. The most important aspect in the calculation models is to find the resultant normal force on idler F_N and the distribution of this force along the zone, i.e., to determine function $q_T(x)$.

Normal loads on idlers (forces F_N, F_{Nm} for middle idler and F_{Ns} for side idler) are due to the influence of the transported material and of the belt. Calculations of normal loads on idlers due to the transported material are based on the bulk solids model (Krause and Hettler 1974), and in such case the resulting load distribution is uniform for the middle idler roll and linearly increasing for the side idler roll. Laboratory tests demonstrated that this is an adequate description of actual conditions. The values of normal loads on idlers are also influenced by the belt. The components of normal forces due to the influence of the belt may be calculated on the basis of empirical relationships described by Kessler (1986). The algorithms for calculating normal loads on idlers with allowance for the parameters of the transported material and for the lateral stiffness of the belt are provided in (Gładysiewicz 2003). Recent tests of loads on a conveyor operated in an open cast lignite mine demonstrated good correlation between the measurement results and the calculations (Kisielewski 2016).

With such assumptions, the equation for rolling resistance in the middle idler is as follows:

$$W_e = 0.926 \cdot \psi_{lab} \cdot \sqrt[3]{\frac{F_{Nm}^4}{D_K^2 \cdot b \cdot c_e}} \qquad (8)$$

and for the side idler:

$$W_e = 0.961 \cdot \psi_{lab} \cdot \sqrt[3]{\frac{F_{Nm}^4}{D_K^2 \cdot b \cdot c_e}}. \tag{9}$$

3. VALIDATION OF THE THEORETICAL MODEL

3.1. CALCULATING THE ELASTIC AND DAMPING PROPERTIES OF THE BELT

Based on laboratory tests of belt samples subjected to continuous cyclical loads, the damping factor ψ_{lab} and the modulus of elasticity E are identified. In the case of textile belts, the samples are prepared from the top cover and from the core, while in the case of steel-cord belts – only from the top cover. A cylindrical sample 50 mm in diameter is placed between parallel steel plates. A special punch is then used to exert harmonic compressive loads with non-zero initial values on the steel plates. During the test, deformations are recorded with a displacement sensor and stresses are recorded with a force sensor. In order to minimize the impact of the surface condition on the measurement results, hysteresis loops were always determined with non-zero initial loads. Hysteresis loops are plotted on the basis of their values. Initial load cycles are not reliable, as the sample of unplasticized belt shows inconsistent deformations. The analysis covers only the loads recorded after more than ten cycles, when deformations stabilize. The hysteresis loop is thus plotted with the omission of the initial cycles (Woźniak 2020).

3.2. COMPARISON OF CALCULATIONS WITH THE MEASUREMENT RESULTS

In order to verify the new theoretical model, the calculation results (with allowance for the laboratory-tested parameters) were compared with the results of measurements performed on a test rig developed at Wroclaw University of Science and Technology (Woźniak et al. 2018; Woźniak 2020). As an example, Fig. 7 shows a comparison for the textile belt (a common belt and a five-ply belt).

The calculation results are underestimated in relation to the measurements performed on the small-scale test rig. The differences are from several to a maximum of 10 N. The accuracy of the correlation between the function and the calculated and measured values are determined by the determination coefficient, which in both cases approximates 1. The correlation between the functions is 0.9986, while the correlation between the points is 0.9885.

The new theoretical model was finally verified by comparing the calculation results with the measurements performed in the lignite mine (Kisielewski 2016). Measuring belt indentation rolling resistances in isolation is not possible in operating conditions, and therefore Fig. 8 shows a relationship between the resistance to motion of an idler set and its mass capacity. The measurement results were compared with the calculation results obtained with the old (Gładysiewicz 2003) and with the new method. The

algorithm proposed in this article includes solely the calculations related to the rolling resistances, and therefore the remaining components of main resistances are calculated
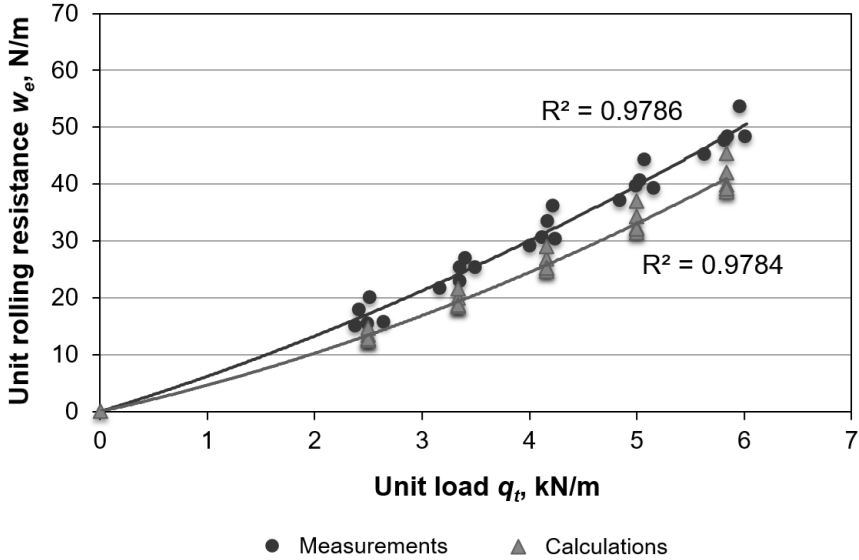

Fig. 7. Comparison of the measurement results with the calculation results
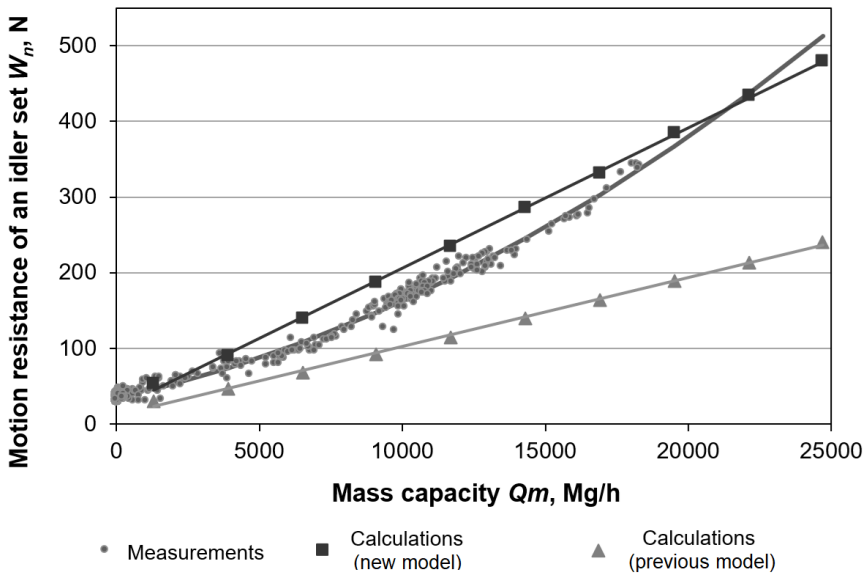of belt indentation rolling resistances for belt T 2000 S

Fig. 8. Comparison of the measured and calculated total resistance of an idler set
as a function of conveyor mass capacity

with the use of the old algorithm (Gładysiewicz 2003). Only the idler rotational resistance was used on the basis of the previous tests of idler characteristics (same that were operated on the actual conveyor) (Kisielewski 2016).

Coefficient of determination for the trend lines describing the measurements and the calculations (Fig. 9) was in each case over 0.99, and therefore the functions which describe the relationship between motion resistances and mass capacity could serve for further analyses, offering high accuracy. More precise calculation of the resistances allows for a more accurate prediction of the energy consumption of belt transportation systems (Hendrik and Katterfeld 2015; Suchorab 2019).

The estimation error (for a maximum capacity of 25 000 Mg/h) was 54.8% in the case of calculations based on the old algorithm and 8.0% for calculations based on the new method.

The calculations involved the values of idler rotational resistances as measured previously in laboratory conditions. The differences between the calculation results and the measurement results may be thus caused by inaccurately calculated remaining components of main resistances, such as belt flexure resistance, flexure resistance of bulk material, sliding resistance of belt on idlers or belt indentation rolling resistance.

4. CONCLUSION

- New relationships have been introduced for calculating belt indentation rolling resistance; they improve the accuracy of calculations in belt conveyors.
- Increased accuracy of the calculations was obtained as a result of:
  - new approach to the parameters which involve the viscoelastic properties of the belt (damping factor ψ and modulus of elasticity E_c),
  - allowance for the non-uniform load distribution along the generator of the idler and for different equations for the middle idler and for the side idlers,
  - deriving a new equation for artificial belt transverse stiffness c_e, which allows for non-uniform load distribution in the belt core.
- Based on theoretical analyses, a method is offered for calculating a damping factor on the basis of a hysteresis loop measured for a continuous harmonic load with non-zero initial value. The loop is not adequate to the belt indentation process, but it is measurable in laboratory conditions.
- The theoretical relationships have been verified in laboratory tests, in which the calculation results were only slightly underestimated in comparison to the measurement results, but the correlation coefficient between the calculation and the measurement results was 0.99.
- Adjusting the size and the drive mechanism in a belt conveyor is a process highly dependent on accurate estimations of the resistances to motion at maximum belt

capacity. The improved method for calculating belt rolling resistances allowed the standard error for maximum capacity to be reduced from 54.8% to 8.0%.

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